

Section 1: Algebra Review

1. Solve $xy + 2x + 1 = y$ for y

2. Factor: $x^2(x - 1) - 4(x - 1)$

3. Solve $\ln(y - 1) - \ln x = x + \ln x$ for y

4. Factor $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

Simplify each expression.

5. $\frac{(x^2)^3 x}{x^7}$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7. $\frac{5(x+h)^2 - 5x^2}{h}$

8. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

Simplify, by factoring first. Leave answers in factored form.

Example:

$$\begin{aligned} \frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} &= \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[4x^2-5x-9-16x-9]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[4x^2-21x-18]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)} \\ &= (x+1)(4x+3) \end{aligned}$$

9. $(x-1)^3(2x-3) - (2x+12)(x-1)^2$

10. $\frac{(x-1)^2(3x-1)-2(x-1)}{(x-1)^4}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

11. $\frac{\sqrt{x+9}-3}{x}$

12. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^4 - 2x^2 = 0$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

15. $\sqrt{x^2 - 9} = x - 1$

16. $|2x - 3| = 14$

17. $x^2 - 2x - 8 < 0$ (answer in interval notation)

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

Solve each system of equations algebraically and graphically.

$$19. \begin{cases} x + y = 8 \\ 2x - y = 7 \end{cases}$$

$$20. \begin{cases} y = x^2 - 3x \\ y = 2x - 6 \end{cases}$$

Section 2: Trigonometry Review

21. Use your knowledge of the unit circle, to evaluate each of the following. You MUST know your unit circle. Leave answers in radical form. Do NOT use your calculator.

a) $\sin 30^\circ$

b) $\cos \frac{2\pi}{3}$

c) $\tan 45^\circ$

d) $\sin\left(-\frac{\pi}{6}\right)$

e) $\tan \pi$

f) $\cos \frac{5\pi}{6}$

g) $\cos(90^\circ)$

h) $\cos \frac{3\pi}{4}$

i) $\cot \frac{\pi}{6}$

j) $\cos^{-1}\left(\frac{1}{2}\right)$

k) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

l) $\tan^{-1}(1)$

Solve each trigonometric equation for $0 \leq x \leq 2\pi$.

22. $\sin x = \frac{\sqrt{3}}{2}$

23. $\tan^2 x = 1$

24. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

25. $2 \sin^2 x + \sin x - 1 = 0$

26. $3 \cos x + 3 = 2 \sin^2 x$

Solve each exponential or logarithmic equation.

27. $5^x = 125$

28. $8^{x+1} = 16^x$

29. $81^{\frac{3}{4}} = x$

30. $8^{-\frac{2}{3}} = x$

31. $\log_2 32 = x$

32. $\log_x \frac{1}{9} = -2$

33. $\log_4 x = 3$

34. $\log_3(x + 7)x = \log_3(2x - 1)$

35. $\log x - \log(x - 3) = 2$

Expand each of the following using the law of logs.

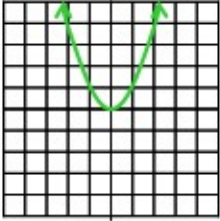
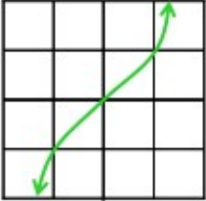
36. $\log_3 5x^3$

37. $\ln \frac{5x}{y^2}$

38. $2\ln\sqrt{y} - \frac{1}{2}\ln y^4 + \ln 2y$

Section 3: Graphing Review

I. Symmetry - Even/Odd Functions

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

To determine algebraically if a function is even, odd, or neither, find $f(-x)$ and determine if it is equal to $f(x)$, $-f(x)$, or neither.

Example: Determine if $f(x) = \frac{4x}{x^2+1}$ is even or odd.

$$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{x^2+1} = -\frac{4x}{x^2+1} = -f(x) \text{ Therefore, } f(x) \text{ is an odd function.}$$

Determine if the following functions are even, odd, or neither.

39. $f(x) = \frac{(x^2)}{x^4+3}$

40. $f(x) = \frac{x}{x+1}$

41. $f(x) = 1 + 3x^2 + 3x^4$

42. $f(x) = 1 + 3x^3 + 3x^5$

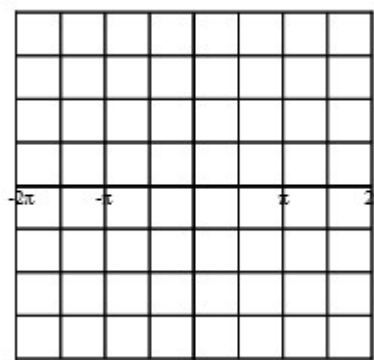
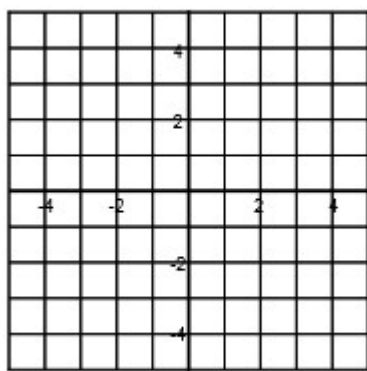
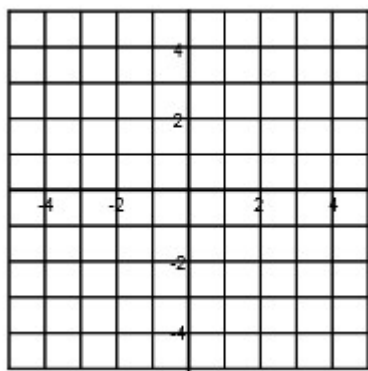
II. Essential Graphs

Sketch each graph. You should know the graphs of these functions.

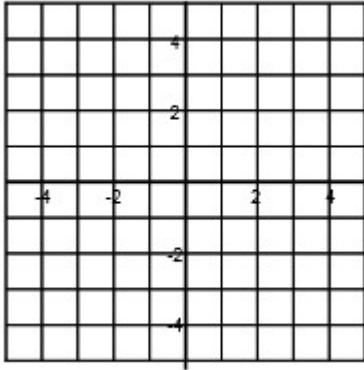
43. $f(x) = \sqrt{x}$

44. $f(x) = x^3$

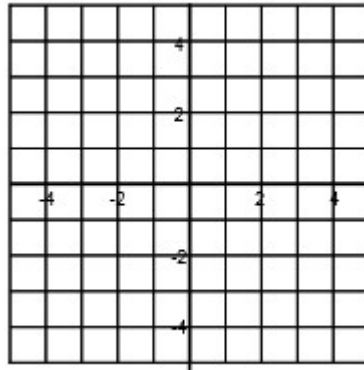
45. $f(x) = \sin x$



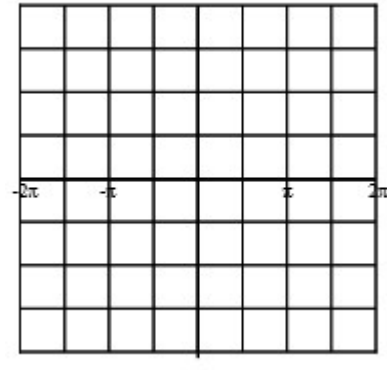
46. $f(x) = e^x$



47. $f(x) = \ln x$



48. $f(x) = \cos x$



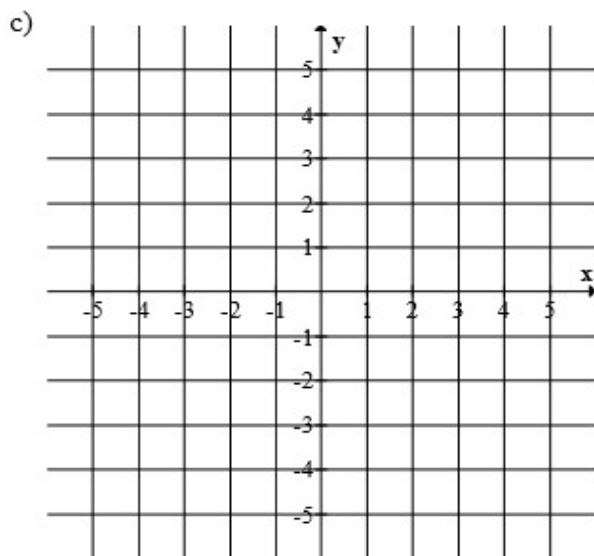
49. For each graph above, state the domain, range, x-intercept(s), y-intercept(s), and any asymptotes - you can write your answers under each graph.

50. For the following functions, find the a) domain, b) range, c) graph, and d) any symmetries.

1. $y = 4 - x^2$

a) _____

b) _____

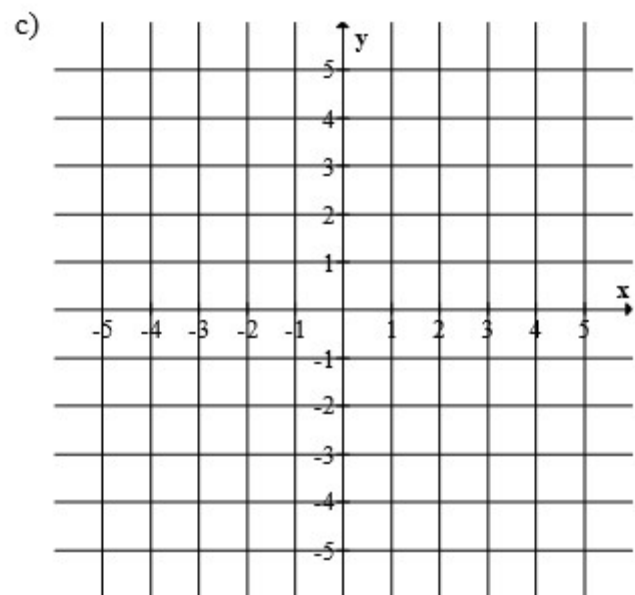


d) _____

2. $y = 2 + \sqrt{x-1}$

a) _____

b) _____

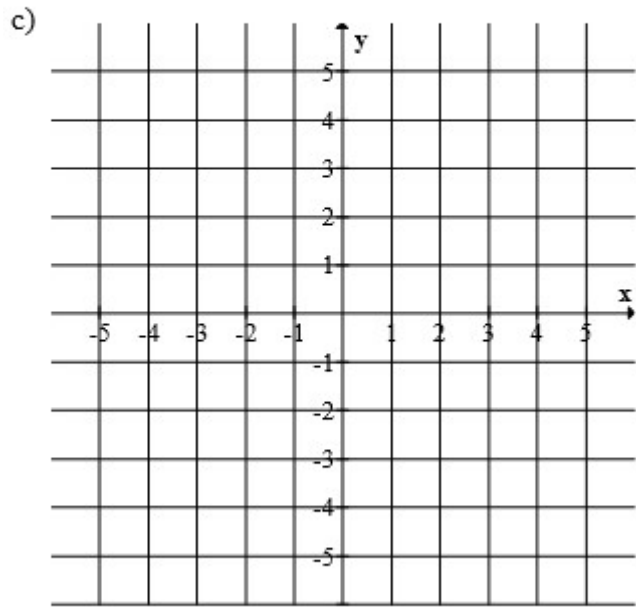


d) _____

3. $y = 1 + \frac{1}{x}$

a) _____

b) _____

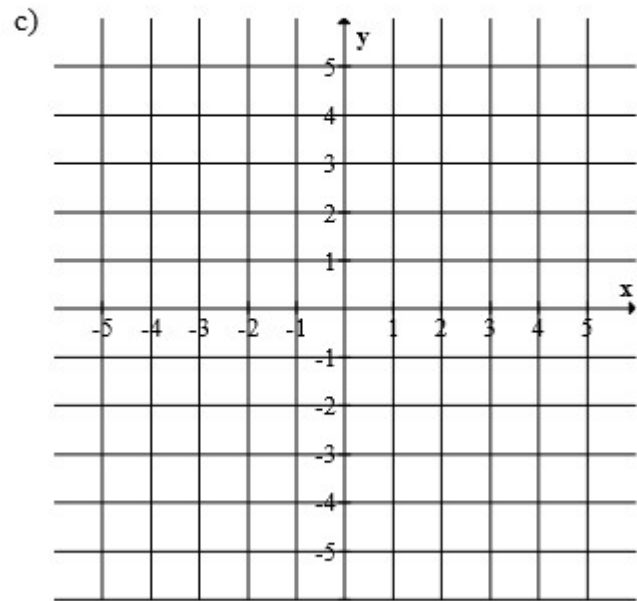


d) _____

4. $y = x^{2/3}$

a) _____

b) _____



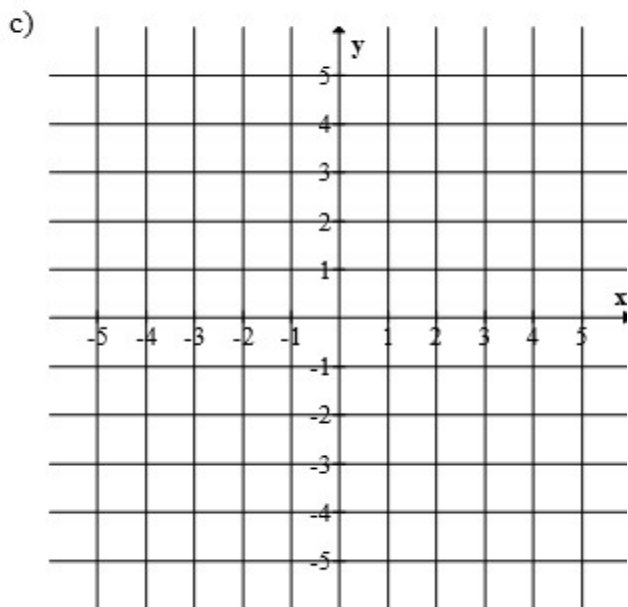
d) _____

51. For the following functions, find the a) domain, b) range, c) graph, and d) any symmetries.

1. $y = -2^x + 3$

a) _____

b) _____

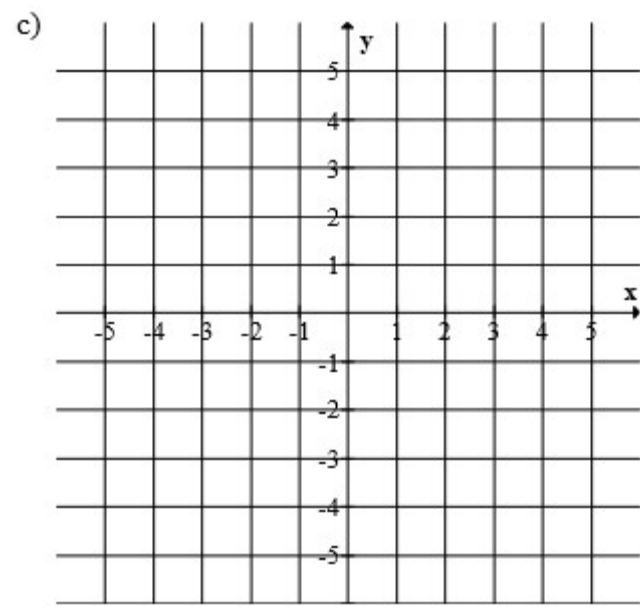


d) _____

2. $y = e^x + 3$

a) _____

b) _____

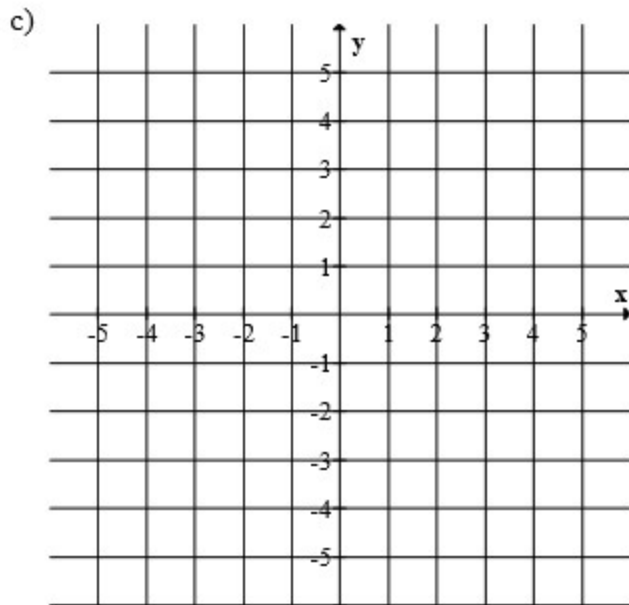


d) _____

3. $y = 3e^{-x} - 2$

a) _____

b) _____

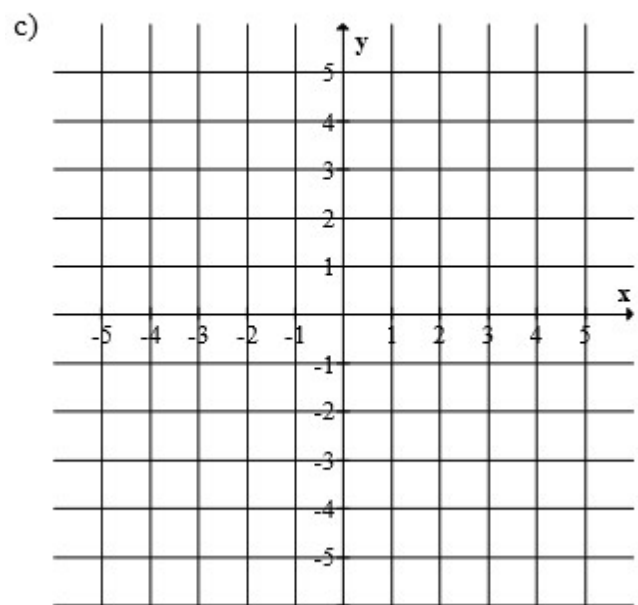


d) _____

4. $y = -2^{-x} - 1$

a) _____

b) _____

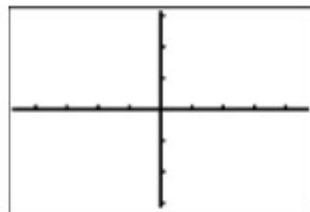


d) _____

Graphing Calculator Skill #1: You should be able to graph a function in a viewing window that shows the important features. You should be familiar with the built-in zoom options for setting the window such as zoom-decimal and zoom-standard. You should also be able to set the window conditions to values you choose.

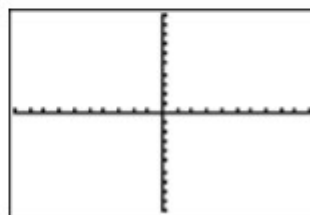
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MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
    
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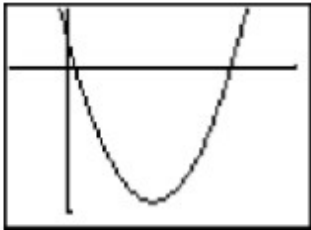


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MEMORY
1:ZBox
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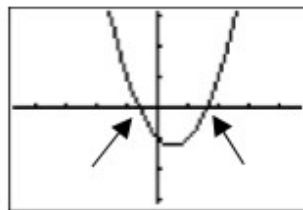
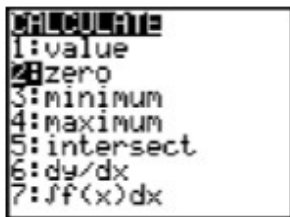
52. Find the appropriate viewing window to see the intercepts and the vertex defined by $y = x^2 - 11x + 10$. Use the window editor to enter the x and y-values.



Window: Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

Graphing Calculator Skill #2: You should be able to graph a function in a viewing window that shows the x-intercepts (also called roots and zeros). You should be able to accurately estimate the x-intercepts to 3 decimal places. Use the built-in zero command on your graphing calculator.

53. Find the x-intercepts of $y = x^2 - x - 1$. Window $[-4.7, 4.7] \times [-3.1, 3.1]$



(Write intercepts as points)

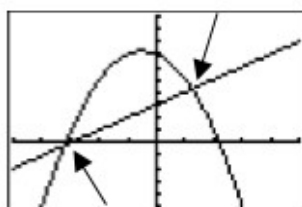
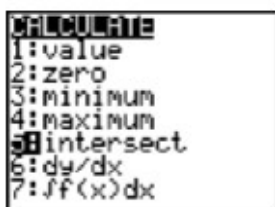
x-intercepts: _____

54. Find the x-intercepts of $y = x^3 - 2x - 1$ _____

Graphing Calculator Skill #3: You should be able to graph two functions in a viewing window that shows the intersection points. Sometimes it is impossible to see all points of intersection in the same viewing window. You should be able to accurately estimate the coordinates of the intersection points to 3 decimal places. Use the built-in intersection command.

55. Find the coordinates of the intersection points for the functions:

$$f(x) = x + 3 \text{ and } g(x) = -x^2 - x + 7$$

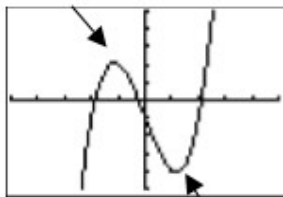
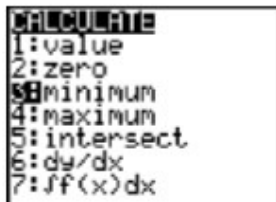


Intersection points: _____

56. Find the coordinates of the intersection points of $f(x) = 4x^2$ and $g(x) = 2^x$

Graphing Calculator Skill #4: You should be able to graph a function and estimate the local maximum and minimum values to 3 decimals. Use the built-in max/min command.

57. Find the maximum and minimum values of the function $y = x^3 - 4x - 1$



(Value means the y-value)

Minimum value: _____

Maximum value: _____

58. Find the maximum and minimum values of the function $y = x^3 - 4x^2 + 4x$

59. Find the x-intercept(s), y-intercept, relative maximum, and relative minimum of $y = x^3 + 2x^2 - 1$

60. Find the coordinates for the points of intersection points for $f(x) = 2x^2 + x - 9$ and $g(x) = -\frac{3}{4}x + 3$

Section 4: Linear Equations

61. Write the equation for the line in both forms given a slope and a point:

a) $m = \frac{2}{3}$ and $P(3,5)$

b) $m = -\frac{4}{5}$ and $P(1,2)$

Point-Slope:

Point-Slope:

Slope-Intercept:

Slope-Intercept:

62. Write the equation of the passing through the given points:

a) $P(2, 2)$ and $Q(4, 2)$

b) $P(3, -2)$ and $Q(3, 7)$

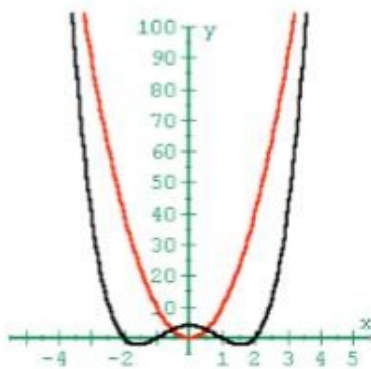
63. The slope of a line is $-\frac{1}{2}$ and the line passes through the points $(2, 5)$ and $(-4, y)$.
Find y .

Section 5: Polynomial Functions

Polynomials : $f(x) = a_n x^n + \dots + a_2 x^2 + a_1 x + a_0$

Studying End Behavior

Even Degree

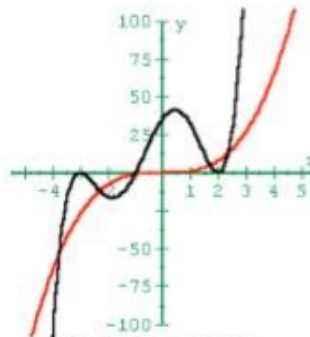


Rises on the Left
Rises on the Right

$$f(x) = x^4 - 5x^2 + 4$$

$$a_n > 0$$

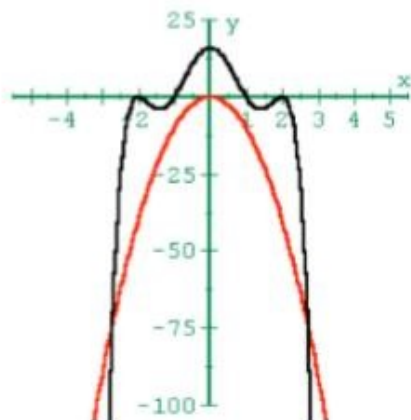
Odd Degree



Falls on the Left
Rises on the Right

$$f(x) = x^5 + 3x^4 - 9x^3 - 23x^2 + 24x + 36$$

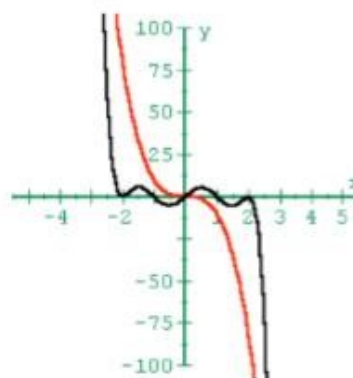
$$a_n > 0$$



Falls on the Left
Falls on the Right

$$f(x) = -x^6 + 9x^4 - 24x^2 + 16$$

$$a_n < 0$$



Rises on the Left
Falls on the Right

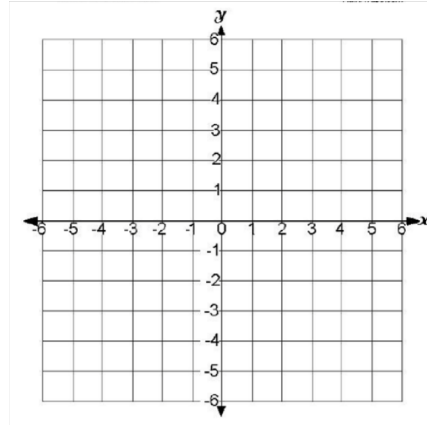
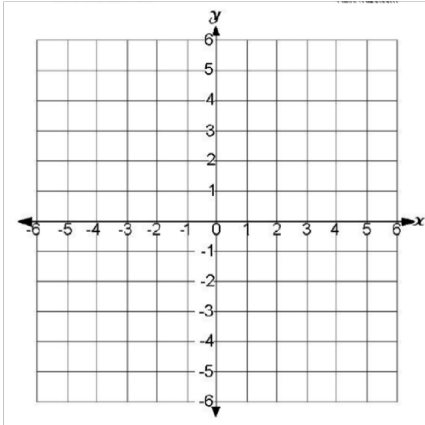
$$f(x) = -x^7 + 9x^5 - 24x^3 + 16x$$

$$a_n < 0$$

64. Sketch a graph of the function without using a calculator. Identify the y-intercept (although it will probably not have drawn to scale on the given grid).

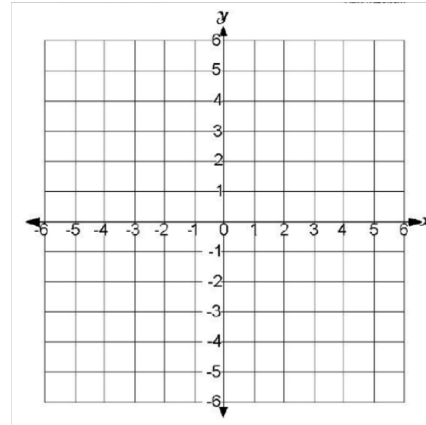
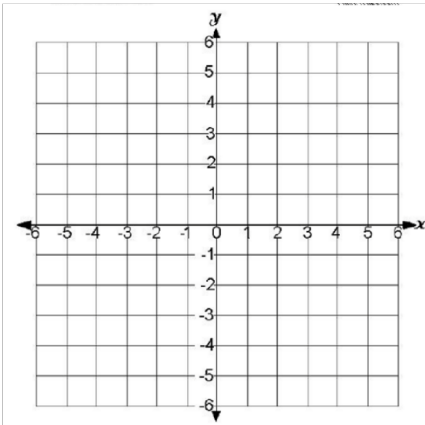
a) $f(x) = -(2x + 7)^3(x - 1)$

b) $g(x) = x^3(x + 4)^2(2x - 5)$

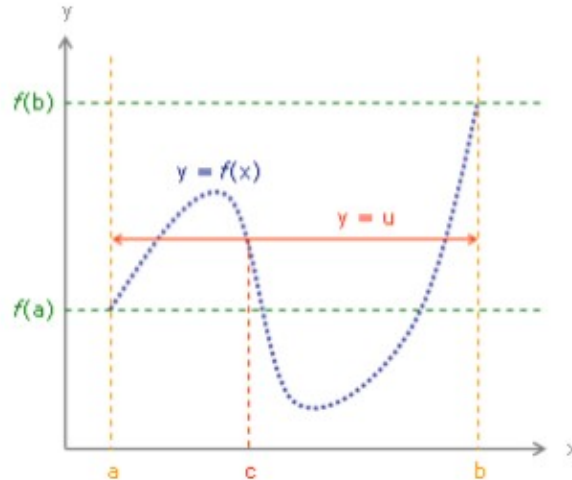


c) $f(x) = -x^2(x - 5)^2(x + 3)$

d) $f(x) = (2x + 1)^3(x - 2)(x - 5)$



The **intermediate value theorem** (IVT) states the following: If the function $y = f(x)$ is continuous on the interval $[a, b]$, and u is a number between $f(a)$ and $f(b)$, then there is a $c \in [a, b]$ such that $f(c) = u$.



Example: Suppose that we want to know if $f(x) = x^4 - 7x^3 - 4x + 8$ is ever zero.

Solution: Since this function is a polynomial, we know that it is continuous everywhere. At $x = -1$, we get $f(-1) = 20$. At $x = 1$, we get $f(1) = -2$. So at the two endpoints of the interval $[-1, 1]$, the function has values of 20 and -2. Therefore, $f(x)$ must take on all values between -2 and 20 as x varies between -1 and 1. In particular, $f(x)$ must take on the value 0 for some x in $[-1, 1]$. The Intermediate Value Theorem (IVT) does not tell us exactly **where** $f(x)$ equals 0, only that it is 0 somewhere on the interval $[-1, 1]$.

65. Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere between -1 and 2 .

66. Use the Intermediate Value Theorem to prove that the equation $x^3 = x + 8$ has at least one solution.

Section 6: Average Rate of Change Review

Definition: Average Speed

Average speed is found by dividing the distance covered by the elapsed time.

$$\frac{\Delta y}{\Delta t} = \frac{\text{total distance traveled}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}}$$

67. Find the average speed of a car that has traveled 350 miles in 7 hours.
68. Suppose $f(1)=2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.
69. The position $p(t)$, in meters, of an object at time t , in seconds, along a line is given by $p(t) = 3t^2 + 1$.
- a) Find the change in position between times $t = 1$ and $t = 3$.
- b) Find the average velocity of the object between times $t = 1$ and $t = 4$.
- c) Find the average velocity of the object between any time t and another time $t + \Delta t$.

Section 7: Parametric Functions

Parametric equations are given below.

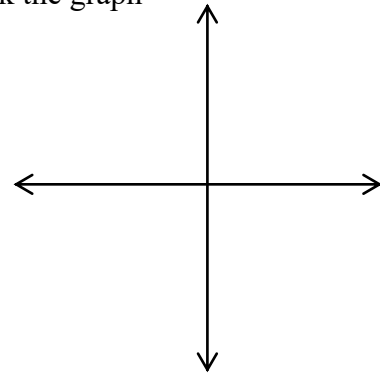
70. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Be sure to define the portion of the graph of the rectangular equation traced by the parametric equations.

a). $x = 4\sin t$, $y = 2\cos t$, $0 \leq t \leq 2\pi$



check the graph

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							

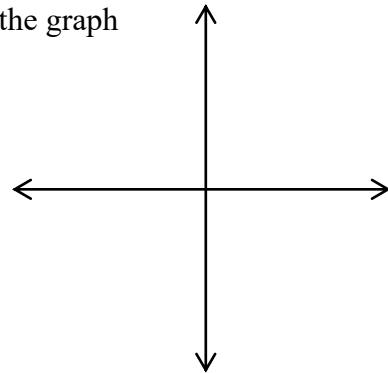


b). $x = 2t - 5$, $y = 4t - 7$, $-2 \leq t \leq 3$



check the graph

t	-2	-1	0	1	2	3
x						
y						



Section 8: Inverse Functions

71. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$

72. If $f(x) = x^3 - 1$, find f^{-1} and verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

73. Discuss the relationship between the domain and range of a function and its inverse.

74. Given the one-to-one function f . The point (a, c) is on the graph of f . Give the coordinates of a point on the graph of f^{-1} .

75. Discuss the relationship between the graph of a function and the graph of its inverse. You can use an example to illustrate your answer.